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The domain walls of gauged maximal supergravities and their M-theory origin

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Abstract: We consider gauged maximal supergravities with CSO(p, q, r) gauge groups and their relation to the branes of string and M-theory. The gauge groups are characterised by n mass parameters, where n is the transverse dimension of the brane. We give the scalar potentials and construct the corresponding domain wall solutions. In addition, we show the higher-dimensional origin of the domain walls in terms of (distributions of) branes. We put particular emphasis on the CSO(p, q, r) gauged supergravities in $D = 9$ and $D = 8$, which are related to the D7-brane and D6-brane, respectively. In these cases, twisted and group manifold reductions are shown to play a crucial role. We also discuss salient features of the corresponding brane distributions.

Keywords: M-Theory, p-branes, Gauge Symmetry, Supergravity Models.

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Contents

1. Introduction

Brane	Vacuum configuration	Gauged SUGRA
M2	$AdS_4 \times S^7$	$D = 4$ SO(8)
M5	$AdS_7 \times S^4$	$D = 7$ SO(5)
D3	$AdS_5 \times S^5$	$D = 5$ SO(6)

Table 1: The branes of the AdS/CFT correspondence.

Due to the AdS/CFT correspondence [1], it has been realised that there is an intimate relationship between certain branes of string and M-theory and lower-dimensional $\mathrm{SO}(n)$ gauged supergravities. The relation is established via a maximally supersymmetric vacuum configuration of string or M-theory, which is the direct product of an AdS space and a sphere. For a p-brane with n transverse directions we are dealing with an $AdS_{p+2} \times S^{n-1}$ vacuum configuration, where $p + n + 1 = 10$ or 11. On the one hand, this vacuum configuration arises as the near-horizon limit of the p-brane in question; on the other hand, the coset reduction over the sphere part leads to the related $\mathrm{SO}(n)$ gauged supergravity in $p + 2$ dimensions which allows for a maximally supersymmetric AdS_{p+2} vacuum configuration. The gauge theory of the AdS/CFT correspondence can be taken at the boundary of this AdS_{p+2} space. All lower-dimensional dilatons are vanishing for this vacuum configuration. This is related to the conformal invariance of the gauge theory. For the branes occurring in the AdS/CFT correspondence, the situation is summarized in table 1.

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There are two ways to depart from the conformal invariance, which both involve exciting some of the dilatons in the vacuum configuration. For the cases given in the table, the scalar potential of the gauged supergravity contains $n-1$ dilatons. By exciting some of these dilatons one obtains a deformed AdS vacuum configuration, which can also be seen as a domain wall. In the AdS/CFT correspondence this corresponds to considering the (non-conformal) Coulomb branch of the gauge theory, see e.g. [2, 3, 4]. Alternatively, one can obtain a non-conformal theory by considering the other branes of string and M-theory, for which there is an extra dilaton present in the scalar potential of the gauged supergravities. In these cases the (maximally supersymmetric) AdS vacuum is replaced by a (non-conformal and half-supersymmetric) domain wall solution. This situation is encountered when one tries to generalise the AdS/CFT correspondence to a DW/QFT correspondence [5, 6, 7].

A natural generalisation is to excite some of the $n-1$ dilatons, leading to the Coulomb branch of the CFT, and the extra dilaton that leads to a non-conformal QFT *simultaneously*. This gives rise to domain wall solutions of $\mathrm{SO}(n)$ gauged supergravities [8] that describe the Coulomb branch of the (non-conformal) QFT. The uplift of these domain walls leads to the near-horizon limit of brane distributions in string and M-theory.

It is the purpose of this paper to extend the analysis of [8] to the case of $\mathrm{CSO}(p,q,r)$ gauged supergravity theories. In doing this we will see that an interesting pattern emerges where all $\mathrm{CSO}(p,q,r)$ gauged supergravities can be treated in a unified way. We will pay particular attention to the branes with a small number of transverse directions, i.e. with $n \leq 3$. As we will see for these cases, we will encounter not only coset reductions on a sphere but also group manifold reductions on S^3 and twisted $\mathrm{SL}(2,\mathbb{R})$ reductions on S^1.

The organisation of the paper is as follows. In section 2 we briefly review some salient features of $\mathrm{CSO}(p,q,r)$ gauged supergravities and their construction by dimensional reduction. The scalar potentials of these theories are separately discussed in section 3. The domain wall solutions of $\mathrm{CSO}(p,q,r)$ gauged supergravities are presented in section 4. The higher-dimensional origin of these solutions as brane solutions in terms of a harmonic over the (flat) transverse space $\mathbb{R}^n$ is discussed in section 5. In section 6 we restrict to the case where the gauge group is $\mathrm{SO}(n)$ or a contracted version thereof and give the higher-dimensional interpretation as brane distributions. We discuss our results in section 7. Finally, in appendix A we perform two limiting procedures on the scalar sector of $\mathrm{CSO}(p,q,r)$ gauged supergravity theories, which are used in the main text.

2. $\mathrm{CSO}(p,q,r)$ gauged supergravities

It has been known for long that certain gauged maximal supergravities with global symmetry groups $\mathrm{SL}(n,\mathbb{R})$ allow for the gauging of the $\mathrm{SO}(n)$ subgroup of the global symmetry. An example is the $\mathrm{SO}(8)$ gauging in four dimensions [9]. Subsequently, it was realised that such gauged supergravities could be obtained by the reduction of a higher-dimensional supergravity over a sphere, with a flux of some field strength through the sphere. For example, the $\mathrm{SO}(8)$ theory can be constructed by the reduction of 11D supergravity over

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D	n	ϕ	Origin	Brane
10	1	$\surd$	Massive IIA [13]	D8
9	2	$\surd$	IIB with SO(2) twist [14]	D7
8	3	$\surd$	IIA on S^2 [15]	D6
7	5	–	11D on S^4 [16, 17]	M5
6	5	$\surd$	IIA on S^4 [18]	D4
5	6	–	IIB on S^5 [11, 12]	D3
4	8	–	11D on S^7 [10]	M2
3	8	$\surd$	IIA on S^7 [19]	F1A
2	9	$\surd$	IIA on S^8 [20]	D0
1	10	$\surd$	$\mathrm{IIB_E}$ on S^9	D-instanton

Table 2: This table indicates the D-dimensional gauged maximal supergravities and the corresponding $(D-2)$-branes, with n the number of mass parameters as well as the number of transverse directions. The third column indicates whether the scalar potential depends on the extra dilaton ϕ; the fourth column gives the higher-dimensional origin of the SO(n) prime examples. The last row corresponds to a (conjectured) reduction of euclidean IIB supergravity on a nine-sphere.

S^7, with magnetic flux of the four-form field strength through the seven-sphere [10]. Other examples are given in table 2.[1]

In addition to SO(n), the global symmetry group SL($n,\mathbb{R}$) has more subgroups that can be gauged. It was found that many more gaugings could be obtained from the SO(n) prime examples by analytic continuation or group contraction of the gauge group [21, 22]. This leads one from SO(n) to the group[2] CSO(p,q,r) with $p+q+r=n$. These gaugings are described in terms of a symmetric matrix Q, which can always be diagonalised as $Q=\mathrm{diag}(q_1,\ldots,q_n)$. Furthermore, the individual q_i's can always be chosen to be ±1 or 0:

$$Q=\begin{pmatrix} \mathbb{I}_p & 0 & 0 \\ 0 & -\mathbb{I}_q & 0 \\ 0 & 0 & 0_r \end{pmatrix}. \tag{2.1}$$

This generalises the SO(n) gauging to $[n^2/4+n]$ different possible gaugings.

The question of the higher-dimensional origin of the CSO gaugings was clarified in [25], where the same operations of analytic continuations and group contractions were applied to the internal manifold. The resulting manifolds are hypersurfaces in $\mathbb{R}^n$ (with cartesian coordinates μ_i) defined by

$$\sum_{i=1}^{n} q_i {\mu_i}^2 = 1\,, \tag{2.2}$$

[1] The S^5 reduction of IIB has not (yet) been proven in full generality. The linearised result was obtained in [11] while the full reduction of the SL($2,\mathbb{R}$) invariant part of IIB supergravity was performed in [12].

[2] The generalisation of SO(p,q) $\equiv$ CSO($p,q,0$) to CSO(p,q,r) is non-trivial for odd-dimensional gauged supergravities; a crucial role is played [23] by the massive self-dual gauge potentials in odd dimensions [24].

where $q_i = 0, \pm 1$ are the diagonal entries of the matrix Q (2.1). The manifold corresponding to (2.2) is denoted by [25]

$$H^{p,q} \circ T^r \,, \tag{2.3}$$

where we use the symbol $\circ$ instead of $\times$ to indicate that the manifold is not a direct product. The corresponding reduction should first be performed over the toroidal part, followed by the hyperbolic manifold $H^{p,q}$. The latter can be endowed with a positive-definite metric, which generically is inhomogeneous [26]; the exceptions are the (maximally symmetric) coset spaces

$$S^{n-1} = H^{n,0} \simeq \frac{\mathrm{SO}(n)}{\mathrm{SO}(n-1)} \,, \qquad H^{n-1} = H^{1,n-1} \simeq \frac{\mathrm{SO}(1,n-1)}{\mathrm{SO}(n-1)} \,, \tag{2.4}$$

i.e. the sphere and the hyperboloid. Generically the spaces $H^{p,q}$ are non-compact; the only exception is the sphere with $q = 0$.

Thus non-compact gauge groups $\mathrm{CSO}(p,q,r)$ with $q \neq 0$ are obtained from reduction over non-compact manifolds, as first suggested in [27]. It can be argued that the corresponding reduction, albeit a non-compactification, is a consistent truncation provided the compact case with $q = 0$ has been proven consistent [25].

We now discuss the special case of $p + q \leq 2$. The manifold (2.2) is then understood as a one-dimensional manifold S^1. It is instructive to consider a chain of contractions, starting from a coset reduction over an $(n-1)$-sphere yielding an $\mathrm{SO}(n)$ gauge group:[3]

$$S^{n-1} \to S^{n-2} \circ S^1 \to \cdots \to S^2 \circ T^{n-3} \to S^1 \circ T^{n-2} \to S^1 \circ T^{n-2} \,. \tag{2.5}$$

From (2.1) and (2.2), the resulting gauge groups should be

$$(n,0,0) \to (n-1,0,1) \to \cdots \to (3,0,n-3) \to (2,0,n-2) \to (1,0,n-1) \,, \tag{2.6}$$

where we have used a short-hand notation $(p,0,r)$ for the group $\mathrm{CSO}(p,0,r)$. The following puzzle arises for the last two links of this chain: the reduction over the torus $S^1 \circ T^{n-2}$ is supposed to lead to the gauge group $\mathrm{CSO}(2,0,n-2)$ and its further contraction $\mathrm{CSO}(1,0,n-1)$, while toroidal reduction naïvely leads to an ungauged theory with trivial gauge group $\mathrm{U}(1)^{n-1}$.

This puzzle is yet more apparent when we consider the simplest case of $n = 2$. In this case the reduction of IIB over S^1 is supposed to lead to the gauge group $\mathrm{SO}(2)$ or its contraction $\mathbb{R}$. The resolution lies in a *twisted* reduction over S^1 [28], making use of the $\mathrm{SL}(2,\mathbb{R})$ duality group of IIB supergravity (see [29, 30, 14, 31, 32, 33]). These twisted reductions give rise to *CSO* gauged supergravity in 9D with $n = 2$. The three different possibilities correspond to twisting with the subgroup $\mathrm{SO}(2)$ of $\mathrm{SL}(2,\mathbb{R})$, the analytic continuation $\mathrm{SO}(1,1)$ and the contraction $\mathbb{R}$, respectively.[4]

[3]Note that the manifold is unchanged in the last contraction.

[4]In the case of SO(2) twisting, the reduction can be given an alternative interpretation as a Kaluza-Klein reduction with a consistent truncation to a higher mode [34].

The reduction Ansatz for a twisted reduction involves an $\mathrm{SL}(2,\mathbb{R})$ transformation of the form [14]

$$\Omega = \exp(Cy)\,, \tag{2.7}$$

where y is the internal coordinate and C is a traceless matrix. Note that general $\mathrm{SL}(n,\mathbb{R})$ twisted reductions [28, 34] give rise to a traceless matrix C. Only when twisting with an $\mathrm{SL}(2,\mathbb{R})$ subgroup of $\mathrm{SL}(n,\mathbb{R})$ can one relate the traceless matrix via

$$C_p{}^q = \epsilon_{pr} Q^{qr}\,, \qquad Q^{qr} = \mathrm{diag}(q_1, q_2)\,, \tag{2.8}$$

to a symmetric matrix Q. Due to (2.8), the traceless matrix C can be related to the parameters q_1 and q_2 of (2.2). The explicit relation between y and the cartesian coordinates μ_i reads

$$\mu_1 = \frac{\sin(\sqrt{q_1 q_2}\, y)}{\sqrt{q_1}}\,, \qquad \mu_2 = \frac{\cos(\sqrt{q_1 q_2}\, y)}{\sqrt{q_2}}\,. \tag{2.9}$$

This explains the relation between twisted reduction and the case $p + q \leq 2$ of (2.2).

The cases $n > 2$ can be treated in a similar way: at the two last links of the above chain of contractions, one must perform an $\mathrm{SL}(2,\mathbb{R})$-twisted reduction. The required $\mathrm{SL}(2,\mathbb{R})$-symmetry is always present for $n \geq 4$, because any reduction over a T^2 factor yields this symmetry. For instance, for $n = 4$ we have:

$$S^3 \to S^2 \circ S^1 \to S^1 \circ T^2\,. \tag{2.10}$$

The reduction over the two-torus gives rise to the $\mathrm{SL}(2,\mathbb{R})$-symmetry that is needed to perform the twisted reduction over the remaining S^1. The same happens for all cases $n \geq 4$. The difference between $(p, q, r) = (2, 0, n-2)$, $(1, 1, n-2)$ and $(1, 0, n-1)$ is the flux of the scalars: the different values correspond to twisting with the subgroups $\mathrm{SO}(2)$, $\mathrm{SO}(1,1)$ and $\mathbb{R}$ of $\mathrm{SL}(2,\mathbb{R})$, respectively.

The $n = 3$ case needs special attention. In this case we are dealing with two-dimensional spaces, e.g. S^2 and H^2, over which one can perform coset reductions. However, by contraction we get

$$S^2 \text{ or } H^2 \;\to\; S^1 \circ S^1\,, \tag{2.11}$$

and we seemingly lack the $\mathrm{SL}(2,\mathbb{R})$-symmetry due to the absence of a T^2 factor. However, the case $n = 2$ has a peculiar feature: the reduction over S^2 or H^2 (i.e. the first link of (2.11)) is only allowed for theories that have an origin in one dimension higher. From the higher-dimensional point of view, the reduction Ansatz over S^2 or H^2 corresponds to a group manifold reduction [6, 35]:

$$\mathcal{G}^{3\mathrm{D}} = (S^2 \text{ or } H^2) \circ S^1 \to S^1 \circ T^2\,. \tag{2.12}$$

Due to the hidden higher-dimensional origin, a two-torus appears on the right hand side, allowing for an $\mathrm{SL}(2,\mathbb{R})$ twisted reduction over the remaining circle.

The higher-dimensional connection corresponds to the relation between M-theory and type-IIA. One can either perform a two-dimensional coset reduction of IIA or a three-dimensional group manifold reduction of 11D to obtain the $\mathrm{SO}(3)$ and $\mathrm{SO}(2,1)$ gauged supergravities in 8D [15, 36]. The contracted versions are obtained by first reducing 11D over T^2 to 9D which produces an $\mathrm{SL}(2,\mathbb{R})$ symmetry in 9D. Next, one applies a twisted reduction form 9D to 8D. Twisting with the subgroups $\mathrm{SO}(2), \mathrm{SO}(1,1)$ and $\mathbb{R}$ of $\mathrm{SL}(2,\mathbb{R})$ leads to $D=8$ gauged supergravities with gauge groups $\mathrm{ISO}(2), \mathrm{ISO}(1,1)$ and Heisenberg, respectively.

In the reduction (2.12) one uses group manifolds of class A of the Bianchi classification, whose structure constants can be written as

$$f_{mn}{}^{p} = \epsilon_{mnq} Q^{pq} \,, \qquad Q^{mn} = \mathrm{diag}(q_1, q_2, q_3) \,. \tag{2.13}$$

Note that the structure constants can only be written in terms of a symmetric matrix Q for *three-dimensional* group manifolds. In the group manifold reduction, the internal metric is described in terms of the Maurer-Cartan 1-forms $\sigma^m = U^m{}_n \, dy^n$, which in turn combine into the structure constants (2.13) of the 8D gauged supergravity and therefore the mass parameters q_i. For more details, see [36].

Reducing from 11D to 10D, one finds a relation between the three-dimensional group manifold reductions (with coordinates y^1, y^2, y^3) and the reductions over the two-dimensional hypersurface (2.2), which boils down to the following expression for the cartesian coordinates

$$\begin{aligned} \mu_1 &= \frac{\sin(\sqrt{q_2 q_3}\, y^2)}{\sqrt{q_1}} \,, \\ \mu_2 &= \frac{\sin(\sqrt{q_1 q_3}\, y^1) \cos(\sqrt{q_2 q_3}\, y^2)}{\sqrt{q_2}} \,, \\ \mu_3 &= \frac{\cos(\sqrt{q_1 q_3}\, y^1) \cos(\sqrt{q_2 q_3}\, y^2)}{\sqrt{q_3}} \,, \end{aligned} \tag{2.14}$$

where $y^{1,2}$ are the two coordinates of the 3D group manifold that remain after reduction over y^3 to 10D.

3. Scalar potentials

We consider truncations of maximal gauged supergravities to the sector with only gravity and the dilatons. The consistency of such truncations have been discussed in e.g. [4, 8]. The corresponding lagrangian in D dimensions is given by the kinetic terms and a scalar potential (due to the gauging):

$$\mathcal{L} = \sqrt{-g} \left(R - \frac{1}{2}(\partial \phi)^2 + \frac{1}{4} \mathrm{Tr}[\partial M \partial M^{-1}] - V \right) , \qquad M = \mathrm{diag}(e^{\vec{\alpha}_1 \cdot \vec{\phi}}, \ldots, e^{\vec{\alpha}_n \cdot \vec{\phi}}) \,. \tag{3.1}$$

The scalars in M are a truncated parametrisation of the scalar coset of the particular maximal supergravity we are considering. In all cases this scalar coset G/H will be of the form

$$M \in \frac{\mathrm{SL}(n,\mathbb{R})}{\mathrm{SO}(n)} \,, \tag{3.2}$$

and it is described by the n vectors $\vec{\alpha}_i = \{\alpha_{iI}\}$, which are weights of $\mathrm{SL}(n,\mathbb{R})$ and satisfy the following relations

$$\sum_{i=1}^{n} \alpha_{iI} = 0 \,, \qquad \sum_{i=1}^{n} \alpha_{iI}\, \alpha_{iJ} = 2\, \delta_{IJ} \,, \qquad \vec{\alpha}_i \cdot \vec{\alpha}_j = 2\, \delta_{ij} - \frac{2}{n} \,, \tag{3.3}$$

with indices $i, j = (1, \ldots, n)$ and $I, J = (1, \ldots, n-1)$. In addition we allow for an extra dilaton ϕ, which would correspond to an extra $\mathbb{R}^+$ factor on the scalar manifold. It is present in some maximal supergravities and absent in others. Explicit information on n, D and ϕ can be found in table 2.

Note that M and ϕ generically do not describe the full scalar coset of maximal supergravities, however, they do constitute the part that is relevant to the CSO gauging and scalar potential. Similarly, the full global symmetry will often be larger than $\mathrm{SL}(n,\mathbb{R})$. Its $\mathrm{SL}(n,\mathbb{R})$ subgroup will generically be the largest symmetry of the lagrangian, however, and is the only part of the symmetry group that is relevant for the present discussion.

The corresponding scalar potential, coming from the gauging of the group $\mathrm{CSO}(p,q,r)$, takes the following form

$$V = e^{a\phi} \Big(\mathrm{Tr}[QMQM] - \frac{1}{2}(\mathrm{Tr}[QM])^2 \Big) \,, \qquad Q = \mathrm{diag}(q_1, \ldots, q_n) \,, \tag{3.4}$$

in terms of n mass parameters q_i. The dilaton coupling a is given by

$$a^2 = \frac{8}{n} - 2\, \frac{D-3}{D-2} \,, \tag{3.5}$$

for the different cases. For later purposes, it is convenient to write the potential V in terms of the superpotential W

$$V = \frac{1}{2}(\vec{\partial} W)^2 + \frac{1}{2}(\partial_\phi W)^2 - \frac{D-1}{4(D-2)}\, W^2 \,, \qquad W = e^{a\phi/2}\, \mathrm{Tr}[QM] \,, \tag{3.6}$$

where $\vec{\partial} W = \partial W / \partial \vec{\phi}$ and $\partial_\phi W = \partial W / \partial \phi$. Note that this form is only possible for dilaton couplings that satisfy (3.5).

In accordance with table 2, a vanishes for $(D, n) = (7, 5)$, $(5, 6)$ and $(4, 8)$, for which the extra dilaton ϕ is absent. The $\mathrm{SL}(2,\mathbb{R})$ twisted reduction of IIB [14, 32] and class A group manifold reduction of 11D [36] yield scalar potentials that coincide with (3.4) for $(D, n) = (9, 2)$ and $(8, 3)$, respectively. In addition, the scalar potential of massive IIA supergravity [13] is also of exactly this form with $(D, n) = (10, 1)$ and is therefore included in table 2.

For the $\mathrm{SO}(n)$ cases, i.e. all $q_i = 1$, the scalar subsector can be truncated by setting $M = \mathbb{I}$. In this truncation, the scalar potential reduces to a single exponential potential

$$V = -\frac{1}{2} n(n-2) e^{a\phi} \,. \tag{3.7}$$

Note the dependence of the sign of the potential on n: it is positive for $n = 1$, vanishing for $n = 2$ and negative for $n \geq 3$. If $a = 0$ (which necessarily implies $n \geq 3$ in $D \geq 4$), the scalar potential becomes a cosmological constant and allows for a fully supersymmetric AdS solution; for this reason, such theories are called AdS supergravities. Theories with $a \neq 0$ are called DW supergravities since the natural vacuum is a domain wall solution.

In the appendix A the dimensional reduction and group contraction (which amounts to setting one q_i to zero) of the scalar potential are discussed. We show that the only effect of these operations is to decrease D or n by one, respectively: the resulting system still satisfies all equations, including (3.5) for the dilaton coupling a, with the new values of the parameters D, n and a. This proves that the scalar subsectors of different gauged supergravities reduce onto each other upon performing dimensional reductions and/or group contractions. We expect this to hold for the full theories as well.

This expectation is supported by the following facts seen from the brane point of view. A p-brane can be reduced in two ways: via a double dimensional reduction (leading to a $(p-1)$-brane in one dimension lower) or a direct dimensional reduction (leading to a p-brane in one dimension lower). It has been pointed out [6] that direct dimensional reduction leads from SO(n) gauged supergravities to the contractions thereof, which fit into the CSO gauged supergravities of [21, 22]. Thus, we get the following relations between operations on the brane and the gauged supergravity:

Brane		Gauged supergravity
direct dimensional reduction	$\Leftrightarrow$	group contraction
double dimensional reduction	$\Leftrightarrow$	circle reduction

4. Domain walls

In this section, we give a unified description for a class of domain wall solutions for gauged supergravities in various dimensions. We consider the following Ansatz for the domain wall with $D-1$ world-volume coordinates $\vec{x}$ and one transverse coordinate y:

$$ds^2 = g(y)^2 d\vec{x}^2 + f(y)^2 dy^2\,, \qquad M = M(y)\,, \qquad \phi = \phi(y)\,. \tag{4.1}$$

The idea is to substitute this Ansatz into the action and write it as a sum of squares, as was done for the conformal cases, i.e. all $q_i = 1$ and $a = 0$, in [37]. Using (3.6), the reduced one-dimensional action can be written as

$$\begin{aligned} S = \int dy\, g^{D-1} f \Bigg[& \frac{D-1}{4(D-2)} \left(\frac{2(D-2)}{fg}\frac{dg}{dy} - W \right)^2 - \frac{1}{2}\left(\frac{1}{f}\frac{d\vec{\phi}}{dy} + \vec{\partial} W \right)^2 + \\ & - \frac{1}{2}\left(\frac{1}{f}\frac{d\phi}{dy} + \partial_\phi W \right)^2 + \frac{1}{f}\frac{dW}{dy} + (D-1)\frac{1}{fg}\frac{dg}{dy} W \Bigg]\,, \end{aligned} \tag{4.2}$$

which is a sum of squares, up to a boundary term. Minimalisation of this action gives rise to the first-order Bogomol'nyi equations

$$\frac{1}{f}\frac{d\vec{\phi}}{dy} = -\vec{\partial} W\,, \qquad \frac{1}{f}\frac{d\phi}{dy} = -\partial_\phi W\,, \qquad \frac{2(D-2)}{fg}\frac{dg}{dy} = W\,. \tag{4.3}$$

Note that one should not expect a Bogomol'nyi equation associated to f since it can be absorbed in a reparametrisation of the transverse coordinate y.

The Bogomol'nyi equations can be solved by the domain wall solution, generalising [4, 8]

$$
\begin{aligned}
ds^2 &= h^{1/(2D-4)} d\vec{x}^2 + h^{(3-D)/(2D-4)} dy^2 \,, \\
M &= h^{1/n} \mathrm{diag}\left(\frac{1}{h_1}, \ldots, \frac{1}{h_n}\right), \qquad e^{\phi} = h^{-a/4} \,,
\end{aligned} \tag{4.4}
$$

written in terms of the n harmonic functions $h_i = 2q_i y + l_i^2$ and their product $h = h_1 \cdots h_n$. Note that this transverse coordinate basis has $\sqrt{-g}g^{tt} = -1$. The functions h_i are necessarily positive since the entries of M are postive. For $q_i > 0$, this implies that y can range from 0 to ∞; if $q_i < 0$, the range of y is bounded from above.

The solution is parametrised by n integration constants[5] l_i. However, if a charge q_i happens to be vanishing, the corresponding l_i can always be set equal to one (by $\mathrm{SL}(n, \mathbb{R})$ transformations that leave the scalar potential invariant). In addition, one can eliminate one of the remaining l_i's by a redefinition of the variable y. Therefore we effectively end up with $p + q - 1$ independent constants.

It should not be a surprise that all scalar potentials of table 2 satisfy the relation (3.5) since these are embedded in a supergravity theory, whose lagrangian "is the sum of the supersymmetry transformations" and therefore always yields first-order differential equations. For this reason, domain wall solutions to the separate terms in (4.2) will always preserve half of supersymmetry. The corresponding Killing spinor is given by

$$
\epsilon = h^{1/(8D-16)} \epsilon_0 \,, \qquad (1 + \Gamma_y)\epsilon_0 = 0 \,, \tag{4.5}
$$

where the projection constraint eliminates half of the components of ϵ_0. An exception is $a = 0$, $q_i = 1$ and $l_i = 0$, in which case the domain wall solution (4.4) becomes a maximally (super-)symmetric Anti-De Sitter space-time in horospherical coordinates. In this case the singularity at $y = 0$ is a coordinate artifact.

5. Higher-dimensional origin

Upon uplifting the domain walls (4.4), one obtains higher-dimensional solutions, which are related to (near-horizon limits of) the 1/2 supersymmetric brane solutions of 11D, IIA and IIB supergravity, as indicated in table 2. Note that the number of mass parameters (and therefore the number of harmonic functions h_i of the transverse coordinate) always equals the transverse dimension of the brane. Thus, in D dimensions, the number of mass parameters is related to the co-dimension of the half-supersymmetric $(D-2)$-brane of IIA, IIB or M-theory.

The metric of the uplifted solution can in all cases be written in the form

$$
ds^2 = H_n^{(2-n)/(D+n-3)} \, dx_{D-1}^2 + H_n^{(D-1)/(D-n-3)} \, ds_n^2 \,, \tag{5.1}
$$

[5] Strictly speaking, it is ${l_i}^2$ rather than l_i that appears as integration constant, allowing for positive and negative ${l_i}^2$. However, one can always take these positive by shifting y, in which case the distinction between l_i and ${l_i}^2$ disappears [4].

where H_n is a harmonic function on the transverse space, whose powers are appropriate for the corresponding D-brane solution in ten dimensions or M-brane solution in eleven dimensions. From the form of the metric, it is therefore seen that the solution corresponds to some kind of brane distribution. For all $q_i = 1$, these solutions were found in [38, 4, 37] for the D3-, M2- and M5-branes and in [8, 39] for the other non-conformal branes. The harmonic function takes the form

$$H_n(y, \mu_i) = h^{-1/2} \left(\sum_{i=1}^{n} \frac{q_i^2 \mu_i^2}{h_i} \right)^{-1} , \tag{5.2}$$

where μ_i are cartesian coordinates, fulfilling (2.2). The transverse part of the metric is given by [4]

$$ds_n^2 = H_n^{-1} h^{-1/2} dy^2 + \sum_{i=1}^{n} h_i \, d\mu_i^2 \, . \tag{5.3}$$

With a change of coordinates, it can be seen that the n-dimensional transverse space is flat [40, 4]

$$z_i = \sqrt{h_i}\mu_i \, , \qquad ds_n^2 = \sum_{i=1}^{n} dz_i dz_i \, . \tag{5.4}$$

The above is easily verified

$$dz_i = h_i^{-1/2} q_i \mu_i \, dy + h_i^{1/2} \, d\mu_i \, , \qquad \sum_{i=1}^{n} dz_i dz_i = \sum_{i=1}^{n} \frac{q_i^2 \mu_i^2}{h_i} \, dy^2 + \sum_{i=1}^{n} h_i \, d\mu_i^2 \, , \tag{5.5}$$

where we have used $\sum_{i=1}^{n} q_i \, \mu_i \, d\mu_i = 0$, which follows by from (2.2). Using (5.2) it is seen that the above agrees with (5.3). Note that one has $\sqrt{-g} g^{ii} = 1$ after the coordinate change to z_i.

The harmonic function H_n specifies the dependence on the n transverse coordinates z_i. The constants l_i parametrise the possible harmonics that are consistent with the reduction Ansatz. The mass parameters q_i specify this reduction Ansatz. Thus, changing a mass parameter q_i changes both the reduction Ansatz and the harmonic function that is compatible with that Ansatz. Sending a mass parameter to zero, e.g. $q_n \to 0$, corresponds to truncating the harmonic function on n-dimensional flat space to

$$H_n(q_n = 0, \; l_n = 1) = H_{n-1} \, , \tag{5.6}$$

i.e. a harmonic function on $(n-1)$-dimensional flat space.

It is difficult to obtain the explicit expression for the harmonic function H_n in terms of the cartesian coordinates z_i (the example of $n = 2$ will be given in (5.13)). Nevertheless, one can show that H_n is indeed harmonic on $\mathbb{R}^n$ for all values of q_i, thus extending the analysis of [38] where $q_i = 1$. The calculation is facilitated by the following definitions

$$A_m = \sum_{i=1}^{n} \frac{q_i^m z_i^2}{h_i^m} \, , \qquad B_m = \sum_{i=1}^{n} \frac{q_i^m}{h_i^m} \, . \tag{5.7}$$

In terms of A_m and B_m we calculate

$$\partial_i H_n = h^{-1/2}\left(-\frac{q_i z_i}{h_i}\frac{B_1}{A_2^2} + 4\,\frac{q_i z_i}{h_i}\frac{A_3}{A_2^3} - 2\frac{q_i^2 z_i}{h_i^2}\frac{1}{A_2^2}\right), \tag{5.8}$$

from which we finally get

$$\sum_{i=1}^{n}\partial_i\partial_i H_n = h^{-1/2}\left(2\frac{B_2}{A_2^2} - 2\frac{B_1A_3}{A_2^3} - 16\frac{A_4}{A_2^3} + 16\frac{A_3^2}{A_2^4} - 2\frac{B_2}{A_2^2} + 16\frac{A_4}{A_2^3} + 2\frac{B_1A_3}{A_2^3} - 16\frac{A_3^2}{A_2^4}\right) = 0\,, \tag{5.9}$$

which proves the harmonicity of H_n on $\mathbb{R}^n$.

The special case of the D6-brane solution (5.1) with $n = 3$ can be uplifted to one dimension higher, where it becomes the 11D Kaluza-Klein monopole:

$$ds^2 = dx_7^2 + H^{-1}\left(dy^3 + \sum_{i=1}^{3} A_i dz_i\right)^2 + \sum_{i=1}^{3} H dz_i{}^2\,, \tag{5.10}$$

where y^3 is the isometry direction of the KK-monopole. The functions $H = H(z_i)$ and $A_i = A_i(z_j)$ are subject to the condition

$$F_{ij} = \frac{1}{2}(\partial_i A_j - \partial_j A_i) = \epsilon_{ijk}\partial_k H\,. \tag{5.11}$$

Generally, this metric is the product of 7D Minkowski space-time and the 4D euclidean Taub-NUT space with isometry direction y^3.

The other special case is the D7-brane with $n = 2$. Its harmonic function reads

$$H_2(y,\mu_i) = \left(\sqrt{h_2}\mu_1{}^2 + \sqrt{h_1}\mu_2{}^2\right)^{-1}. \tag{5.12}$$

In this case, it is straightforward (though perhaps not very insightful) to perform the coordinate transformation to z_i, which yields:

$$H_2(z_1,z_2) = \left(\frac{\alpha z_1{}^2 + \beta z_2{}^2 + \gamma(z_1{}^2 + z_2{}^2)}{2\gamma^2}\right)^{1/2}, \tag{5.13}$$

with the definitions

$$\begin{aligned} \alpha &= q_1q_2(z_1{}^2 + z_2{}^2) + q_1 l_2^2 - q_2 l_1^2\,, \\ \beta &= q_1q_2(z_1{}^2 + z_2{}^2) - q_1 l_2^2 + q_2 l_1^2\,, \end{aligned} \qquad \gamma = \sqrt{\frac{1}{2}(\alpha^2+\beta^2) + q_1q_2(z_1{}^2 - z_2{}^2)(\alpha-\beta)}\,. \tag{5.14}$$

Indeed, it can be checked that this function is harmonic with respect to flat (z_1, z_2)-space for all values of q_i and l_i.

The special SO(2)-case with $q_1 = q_2 = 1$ and $l_1 = l_2$ leads to the trivial harmonic function

$$H_2 = 1\,. \tag{5.15}$$

Actually, for this case the 9D scalar potential is vanishing, $V = 0$, see also (3.7). Although the scalar potential is vanishing, the corresponding superpotential is non-vanishing: $W \neq 0$. The corresponding 9D solution is uplifted to a 10D conical spacetime. For more information, see [32].

As another example, the $\mathbb{R}$-case with charges $(q_1, q_2) = (1, 0)$ leads to

$$H_2(q_2 = 0) = \frac{|z_1|}{l_2}\,, \tag{5.16}$$

which is a harmonic function in a one-dimensional transverse space, in agreement with (5.6). This case describes the "circular" D7-brane of [29] that is T-dual to the D8-brane. Note that the previous case with $q_1 = q_2$ has no manifest T-dual picture.

6. Brane distributions for SO(n) harmonics

In this section we restrict ourselves to the SO(n) cases and their group contractions.

Since the harmonic function H_n depends on the angular variables in addition to the radial, the uplifted solution will in general correspond to a distribution of branes rather than a single brane. For $D < 9$ and $q_i = 1$ (i.e. the SO(n) cases[6] with $n \geq 3$) this means that the harmonic function can be written in terms of a charge distribution σ as follows [4, 8]

$$H_n(\vec{z}) = \int d^n z' \frac{\sigma(\vec{z}\,')}{|\vec{z} - \vec{z}\,'|^{n-2}}\,, \qquad n \geq 3\,, \tag{6.1}$$

and since H_n appears without an integration constant, the distributions will actually be a near-horizon limit of the brane distribution.

It turns out that the distributions are given in terms of higher dimensional ellipsoids [2, 4]. The dimension of these ellipsoids are given in terms of the number m of non-vanishing constants l_i. It is convenient to define

$$x_m = 1 - \sum_{i=1}^{m} \frac{z_i^2}{l_i^2}\,, \qquad \vec{l} = (l_1, \ldots l_m, 0, \ldots, 0)\,, \tag{6.2}$$

where the last $n - m$ constants l_i are vanishing. Starting with the case $m = n - 1$, we have a negative charge[7] distributed inside the ellipsoid and a positive charge distributed on the boundary [3, 4]:

$$\sigma_{n-1} \sim \frac{1}{l_1 \cdots l_{n-1}} \Big(- x_{n-1}^{-3/2} \Theta(x_{n-1}) + 2\, x_{n-1}^{-1/2} \delta(x_{n-1}) \Big) \delta^{(1)}(z_n)\,. \tag{6.3}$$

Upon sending l_{n-1} to zero, the charges in the interior of the ellipsoid cancel, leaving one with a positive charge on the boundary of a lower dimensional ellipsoid:

$$\sigma_{n-2} \sim \frac{1}{l_1 \cdots l_{n-2}}\, \delta(x_{n-2})\, \delta^{(2)}(z_{n-1}, z_n)\,. \tag{6.4}$$

Next, the contraction of more constants will yield brane distributions over the inside of an ellipsoid. The distribution $\sigma(z_i)$ is then a product of a delta-function and a theta-function and the branes are localised along $n - m$ coordinates and distributed within an m-dimensional ellipsoid, defined by $x_m = 0$. For $1 \leq m \leq n-3$ non-zero constants, one has

$$\sigma_m \sim \frac{1}{l_1 \cdots l_m}\, x_m^{(n-m-4)/2}\, \Theta(x_m)\, \delta^{(n-m)}(z_{m+1}, \ldots, z_n)\,. \tag{6.5}$$

[6] Note that the group contractions are included by taking some $q_i = 0$, using (5.6).

[7] However, these negative charges might be pathological, since the tension will also be negative [3, 41].

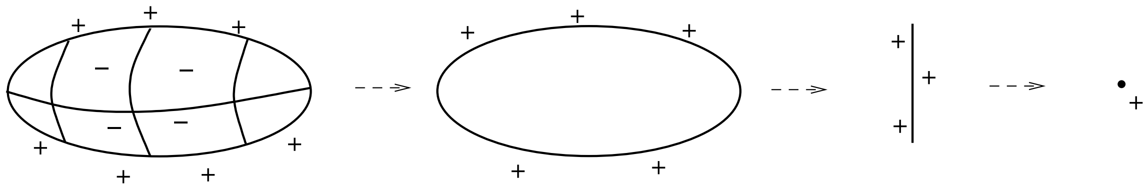

Figure 1: The distributions of D5-branes with three, two, one and zero non-vanishing l_i's, respectively.

Finally, one is left with all constant l_i vanishing, in which case the distribution has collapsed to a point and generically reads

$$\sigma_0 = \delta^{(n)}(z_1, \dots, z_n) \,, \tag{6.6}$$

i.e. we are left with (the near-horizon limit of) a single brane. All these distributions satisfy

$$\sigma_{m-1} = \delta(z_m) \int \sigma_m \,, \tag{6.7}$$

consistent with the picture of distributions that collapse the z_m-coordinate upon sending l_m to zero. The case of D5-branes[8] is illustrated in figure 1.

The uplift and the corresponding distributions were found in [4] for the D3-, M2- and M5-branes. The extension to the other branes was treated in [8]. Special cases have also been studied in [3, 41]. For certain cases, i.e. when the l_i's are pairwise equal, the distributions above are related to the extremal limit of rotating branes [2, 42], and the l_i's then correspond to rotation parameters.

The special case of the D6-brane distributions (with all $q_i = 1$) was first discussed in [8]. This splits up in three separate possibilities, with $m = 2, 1$ or 0. The first distribution σ_2 consists of positive and negative densities and is given by the general formula (6.3). Upon sending l_2 to zero, this collapses to

$$\sigma_1 \sim \frac{1}{l_1}\, \delta\left(1 - \frac{{z_1}^2}{{l_1}^2}\right) \delta^{(2)}(z_2, z_3) \,. \tag{6.8}$$

This is a distribution at the boundary of a one-dimensional ellipse, i.e. it is localised at the points $z_1 = \pm l_1$. For this reason, the corresponding harmonic function is given by

$$H_3(\vec{z}, l_1) = \frac{1}{2((z_1 - l_1)^2 + {z_2}^2 + {z_3}^2)^{1/2}} + \frac{1}{2((z_1 + l_1)^2 + {z_2}^2 + {z_3}^2)^{1/2}} \,, \tag{6.9}$$

i.e. the near-horizon limit of the double-centered D6-brane. Note that this is the only non-trivial case (i.e. with at least one l_i non-vanishing) where we get a discrete rather than a continuous distribution. Upon sending l_1 to zero, the brane distribution σ_0 collapses to a point, as given in (6.6). Indeed, the harmonic function becomes

$$H_3 = \frac{1}{|\vec{z}|} \,, \tag{6.10}$$

i.e. the near-horizon limit of the single-centered D6-brane with SO(3)-isometry. The different distributions of D6-branes are shown in figure 2.

[8]For remarks about the corresponding 7D gauged supergravity, see the discussion.

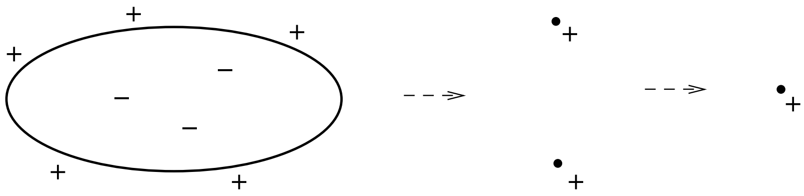

Figure 2: The distributions of D6-branes with two, one and zero non-vanishing l_i's, respectively.

These explicit harmonic functions also give rise to special cases when uplifted to the 11D Kaluza-Klein monopole (5.10). The first consists of all $q_i = 1$ and only $l_1 \neq 0$, in which case the harmonic function corresponds to the near-horizon limit of the two-centered solution. The Taub-NUT geometry degenerates to the Eguchi-Hanson geometry in this case [43]. Secondly, the parameter l_1 can be sent to zero, yielding the near-horizon limit of a single-centered Kaluza-Klein monopole. In this case, the 4D euclidean space becomes (locally) flat.

In the other special case, with $n = 2$ and $q_1 = q_2 = 1$, the IIB solution can be understood as a distribution of D7-branes. Without loss of generality we take $l_2 = 0$. Now (6.1) does not apply and the harmonic function will instead be given by

$$H_2(\vec{z}) = 1 + \int dz_1' dz_2' \, \sigma_1(z_1', z_2'; l_1) \log((z_1 - z_1')^2 + (z_2 - z_2')^2) \,, \tag{6.11}$$

with the D7-brane distribution

$$\sigma_1 = \frac{1}{2\pi l_1} \left[-\left(1 - \frac{z_1'^{\,2}}{l_1^2}\right)^{-3/2} \Theta\left(1 - \frac{z_1'^{\,2}}{l_1^2}\right) + 2\left(1 - \frac{z_1'^{\,2}}{l_1^2}\right)^{-1/2} \delta\left(1 - \frac{z_1'^{\,2}}{l_1^2}\right) \right]. \tag{6.12}$$

Note that this distribution consists of a line interval of negative D7-brane density with positive contributions at both ends of the interval. Both positive and negative contributions diverge but these cancel exactly:

$$\int dz_1' dz_2' \, \sigma_1(z_1', z_2') = 0 \,, \tag{6.13}$$

i.e. the total charge in the distribution (6.12) vanishes.

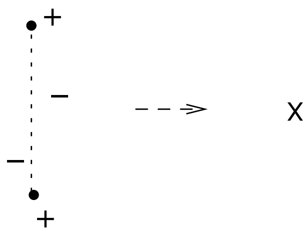

Figure 3: The distributions of D7-branes with one and zero non-vanishing l_i's, respectively. The cross indicates the conical singularity of the locally flat space-time.

The parameter l_1 of the general SO(2) solution can be set to zero. This corresponds to a collapse of the line interval to a point, as can be seen from (6.12). However, due to the fact that the total charge vanishes, this leaves us without any D7-brane density:

$$\sigma_0 = 0 \,. \tag{6.14}$$

This is consistent with the fact that the general harmonic function (5.13) equals the one for the SO(2) case with $l_1 = l_2 = 0$. The D7-brane distributions are shown in figure 3.

Thus, the two-dimensional SO(2) harmonic function (6.11) of the D7-brane differs in two important ways from the generic SO(n) harmonic function with $n > 2$. Firstly, the total charge distribution of D7-branes vanishes, while it adds up to a finite and positive number in the other cases. Secondly, but not unrelated, one needs to include a constant in the harmonic function (6.11) in terms of the distribution. In the generic cases this constant was absent, corresponding to the near-horizon limit of these branes. For the D7-brane, the concept of a near-horizon limit is unclear, as we comment upon in the discussion.

7. Discussion

In this paper we have discussed the construction of CSO gauged supergravities via dimensional reduction over a coset manifold, a group manifold or with a twist. In addition, half-supersymmetric domain wall solutions were constructed. The uplifting of these solutions to string and M-theory leads to brane solutions characterized by a harmonic function H_n on the flat transverse space $\mathbb{R}^n$. At this point our results apply for arbitrary values of the charges q_i thereby generalizing the case of all $q_i = 1$ [4, 8]. The rewriting of the harmonic function H_n in terms of an integral representation with a brane distribution function $\sigma_m(\vec{z})$ was discussed for the SO(n) case and its group contractions, i.e. all $q_i \geq 0$, where special emphasis was put on the SO(2) and SO(3) cases. It would be interesting, if possible, to extend this part of the discussion to arbitrary values of the charges.

We would like to comment on the special features of the SO(2) harmonic function corresponding to the D7-brane that we encountered in section 6. The near-horizon limit of D-branes does not yield a separated spherical part in Einstein frame; for this one needs to go to the so-called dual frame, in which the tension of the brane is independent of the dilaton

$$g_{\mu\nu}^{\text{dual}} = \exp\left(\frac{(3-p)}{2(p-7)}\,\phi\right) g_{\mu\nu}^{\text{Einstein}}\,. \tag{7.1}$$

In the dual frame, the near-horizon geometry of all D-branes with $p \leq 6$ reads[9] $AdS_{p+2} \times S^{n-1}$. Clearly, this formula does not hold for the D7-brane; a related complication is the fact that the dual object is the D-instanton, which lives on a euclidean space.

One of the themes of this paper has been that for every brane there is a corresponding gauged supergravity. Sofar, however, we have not treated all the branes of string and M-theory. For one thing, we did not explicitly include the NS5A-brane in table 2. Since this brane is the direct dimensional reduction of the M5-brane, it leads to the $D = 7$ ISO(4) gauged supergravity of [18]. Only the SO(4) symmetries are linearly realized, the other four symmetries occur as Stueckelberg symmetries. A similar story holds for the D2-brane which is the direct dimensional reduction of the M2-brane. Furthermore, we did not consider the IIB doublets of NS5B/D5- and F1B/D1-branes. The associated theories are the reduction of IIB over S^3 or S^7 with an electric or magnetic flux of the NS-NS/R-R three form field-strength [45, 35]. For the $D = 3$ SO(8) theories corresponding to the IIB strings (which are different than the F1A result), see [19]. The five-brane cases are supposed to lead to new $D = 7$ SO(4) gauged supergravities, which might be related to the theories constructed in [46].

[9] Except for the case $p = 5$, which has Minkowski$_7$ rather than AdS_7 [44].

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Finally, we would like to mention that there are also gauged maximal supergravities which are not of the CSO class considered in this paper, see e.g. [47, 48, 49]. In addition, there are gauged maximal supergravities that do not have an action but only have field equations [50, 51]; such theories do not allow for domain wall solutions, however. It would be interesting to investigate the structure of such gaugings and to establish whether the absence of domain wall solutions is a general feature of these theories.

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A. Group contraction and dimensional reduction

We would like to consider two operations on the scalar sector of the CSO gauged supergravity. The first operation corresponds to a contraction of the CSO gauge group and corresponds to setting one mass parameter equal to zero. For concreteness it is taken to be the last one: $q_i = (q_p, 0)$, where we have split up $i = (p, n)$ and $p = 1, \ldots, n-1$. The superpotential now reads

$$W = e^{a\phi/2} \sum_p q_p \, e^{\vec{\alpha}_p \cdot \vec{\phi}} = e^{a\phi/2 + \vec{\beta}\cdot\vec{\phi}} \sum_p q_p \, e^{\vec{\beta}_p \cdot \vec{\phi}} \,, \tag{A.1}$$

where we have chosen to split off an overall part $\vec{\beta}\cdot\vec{\phi}$ according to $\vec{\alpha}_p = \vec{\beta} + \vec{\beta}_p$. A convenient choice for $\vec{\beta}$ is

$$\vec{\beta} = -\frac{1}{n-1}\vec{\alpha}_n = \left(0, \ldots, 0, \frac{1}{\sqrt{n(n-1)/2}}\right) . \tag{A.2}$$

This corresponds to the scalar coset split

$$M = \begin{pmatrix} e^{\vec{\beta}\cdot\vec{\phi}}\tilde{M} & 0 \\ 0 & e^{-(n-1)\vec{\beta}\cdot\vec{\phi}} \end{pmatrix} , \qquad \tilde{M} = \mathrm{diag}(e^{\vec{\beta}_1\cdot\vec{\phi}}, \ldots, e^{\vec{\beta}_{n-1}\cdot\vec{\phi}}) \,, \tag{A.3}$$

where the weight vectors $\vec{\beta}_p$ are subject to the reduction of (3.3):

$$\sum_p \beta_{pI} = 0 \,, \qquad \sum_p \beta_{pI}\,\beta_{pJ} = 2\,\delta_{IJ} \,, \qquad \vec{\beta}_p \cdot \vec{\beta}_q = 2\,\delta_{pq} - \frac{2}{n-1} \,, \tag{A.4}$$

while the last component of all vectors $\vec{\beta}_p$ vanishes: $\beta_{pn} = 0$. Therefore, the contracted superpotential (A.1) only depends on the smaller coset $\mathrm{SL}(n-1, \mathbb{R})/\,\mathrm{SO}(n-1)$. Also note that the overall dilaton coupling has changed due to the contraction. For the scalar potential, this will amount to $a\phi+2\vec{\beta}\cdot\vec{\phi}$ instead of $a\phi$. After a change of basis, corresponding to an SO$(n+1)$ rotation in $(\phi,\vec{\phi})$-space, this takes the form $\tilde{a}\tilde{\phi}$ with

$$\tilde{a}^2 = a^2 + 4\vec{\beta}\cdot\vec{\beta} = \frac{8}{n-1} - 2\,\frac{D-3}{D-2} > a\,, \tag{A.5}$$

which is exactly the original relation (3.5) with n decreased by one. It should be clear that this contraction can be employed several times, each time reducing n by one.

The second operation we wish to perform corresponds to dimensionally reducing the scalar sector. We take trivial Ansätze[10] for the scalars, $\hat{M}=M$ and $\hat{\phi}=\phi$, and the usual Ansatz for the metric (obtaining Einstein frame with a canonically normalised Kaluza-Klein scalar φ in the lower dimension):

$$d\hat{s}^2 = e^{2\gamma\varphi}ds^2 + e^{-2(D-3)\gamma\varphi}dz^2\,, \qquad \gamma^2 = \frac{1}{2(D-2)(D-3)}\,, \tag{A.6}$$

where we have truncated the Kaluza-Klein vector away. The resulting scalar potential is of the same form (3.4), but again the dilaton coupling has changed: the factor $a\phi$ is replaced by $a\phi+2\gamma\varphi$. After a field redefinition, this corresponds to $\tilde{a}\tilde{\phi}$ with

$$\tilde{a}^2 = a^2 + 4\gamma^2 = \frac{8}{n} - 2\,\frac{D-4}{D-3} > a\,, \tag{A.7}$$

which is exactly the original relation (3.5) with D decreased by one. Again, dimensional reduction can be performed any number of times, reducing D by one at each step.

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